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Free convection effects on horizontal magnetohydrodynamic channel flow with variable viscosity

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An analysis of mhd channel flow with heat transfer under the influence of the crossed fields, buoyancy forces and variable viscosity is carried out. Solutions for the velocity, current density, magnetic field temperature are derived and are shown graphically. The numerical values of the skin-friction and the Nusselt numbers are entered in tables. It is observed that in constant viscosity case, the skin-friction and Nu increases with more heating of the channel plates and decreases with more heating of the channel plates and decreases with more coolng of the channel plates. In case of variable viscosity, with more cooling of the plates, there is a tendency of separation at the lower plate. In the presence of bouyancy forces, the skin-friction decreases. In a mhd generator, an increase in the loading parameter or due to more heating of the plates, the skin-friction increases.

1. INTRODUCTION

MHD channel flows have been discussed extensively in recent years for their wide applications in technology. The heat transfer aspect of such flows has also been studied in case of fully developed flows by Siebel (1958), Gershuni & Zukhovitskii (1958), Sutton and Sherman (1965), Soundalgekar (1968), whereas Nigam & Singh (1960), Perlmutter & Siegel (1961), Erickson *et al* (1966) have discussed it in the entrance region of channel flow. In all these investigations, due to hori-zontal flow, the effects of the buoyancy forces have not been taken into acconnt.

This is due to the assumption that the buoyancy forces are quite negligible in horizontal flows. This is not always true. It was shown by Sparrow *et al* (1957) that in case of horizontal flows of low Prandtl number fluids, the buoyancy forces cannot be neglected as they significantly affect the flow field. Independently, it was also shown by Gill and Casal (1962) that the effects of the buoyancy forces are significantly important in case of the horizontal flows of the low Prandtl number fluids. Gill & Casal also discussed the effects of the variable viscosity on the horizontal channel flow between two parallel infinite plates.

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All the low Prandtl number fluids are electrically conducting and hence their flow is affected by transversly applied magnetic field. This property has been utilised profitably in magnetohydrodynamic channel flows. But all attempts to analyse the mhd flows were without considering the buoyancy forces. This led Gupta (1969) to investigate the effects of the gravitational field on the horizontal mhd channel flow of electrically conducting, viscous incompressible fluid for open circuit case. Gupta, however, assumed the viscosity to be constant. Recently, the effects of variable viscosity on horizontal mhd channel flow were discussed by Soundalgekar & Haldavnekar (1973).

The study of mhd channel flows is important from practical point of view as its works as a generator or an accelerator depending upon the value of the loading parameter $e = E/VB$ where V, E, B are respectively the fluid velocity, electric field strength and the magnetic field intensity. Moffatt (1963) has shown that, i) $e > 1$ corresponds to a mhd generator, ii) $e > 1$ corresponds to a mhd accelerator and iii) $e = 0$ or 1 corresponds to short or open-circuited case. Hence, in order to study the mhd channel flows from practical point of view, it is necessary to consider the effects of i) crossed-fields ii) gravitational field and iii) the variable properties of the fluids. It is now the object of the present investigation to study the effects of these forces on horizontal mhd channel flow when the viscosity of the fluid is a linear function of the temperature and the temperature of the plates also varies linearly. For electrically conducting fluids, such an assumption does give good results. In section 2, the problem is posed in a suitable manner and the expressions for the velocity and temperature are first derived under constant property assumptions. Then for the case of variable viscosity, neglecting dissipation and the source terms in the energy equation, under a suitable transformation, the problem is completely solved and expressions for the velocity profiles, current density, magnetic field and temperature profiles are plotted on graphs and the numerical values of the skin friction and the Nusselt number are entered in the tables.

In section 3, the conclusions are presented wherein the effects of heating or cooling of the plates, the buoyancy forces and the loading parameter are described.

2. Mathematical Analysis

Here a steady, laminar flow of a viscous, incompressible, electrically conducting fluid between two infinite and non-conducting plates in the x and z directions is assumed. The x-axis is chosen along the lower plate in the direction of the flow and the y-axis is chosen normal to it. A uniform magnetic field is assumed to be applied parallel to the y-axis. The electrical conductivity of the fluid medium is assumed to be a constant scalar quantity. For steady flow, the displacement current vanishes identically.

Under these conditions, considering the buoyancy forces, the fully developed mhd flow is governed by the following equations

$$-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u_1}{\partial y^2}-j_z B_0=\rho g_x, \tag{1}$$

$$-\frac{\partial p}{\partial y}+j_z B_x=\rho g_y \tag{2}$$

Here u_1 is the velocity component, ρ the density, μ the viscosity, p the pressure, j_z the current density, $(B_x, B_0, 0)$ the components of the magnetic field intensity and g_x, g_y are the components of the gravitational force. Neglecting Hall current, the Ohm's law is

$$j_z=\sigma(E_z+u_1 B_0), \qquad \ldots(3)$$

where E_z is the applied electric field. The equation of state is

$$\rho=\rho_0[1-\beta(t-t_0)] \tag{4}$$

where β is the coefficient of expansion, t the temperature and t_0, ρ_0 are the initial temperature and density, assumed constant.

If the temperature varies linearly in the direction of flow, then the energy equation is

$$\rho c_p u_1\frac{\partial t}{\partial x}=k\frac{\partial^2 t}{\partial y^2}+\mu\left(\frac{\partial u_1}{\partial y}\right)^2+\frac{j_z^2}{\sigma}+Q, \tag{5}$$

where the last three terms represent respectively the heat due to viscous dissipation, Joule dissipation and the constant heat source. Also c and k are respectively the specific heat and the thermal conductivity of the fluid. In view of the x-axis being perpendicular to the gravity force, the terms ρg_x in eq. (1) is identically zero. Then eliminating ρ between eqs. (2) and (4), we get

$$\frac{1}{\rho_0}\frac{\partial p}{\partial y}+\frac{1}{\rho_0}j_z B_x=g_y[1-\beta(t-t_0)]. \qquad \ldots(6)$$

In fully developed flow, all the physical variables, except the pressure, are functions of y only. Hence differentiating (6) with respect to x, we obtain

$$-\frac{1}{\rho_0}\frac{\partial^2 p}{\partial x\partial y}=-\beta g_y\frac{\partial(t-t_0)}{\partial x}. \tag{7}$$

Differentiating (1) with respect to y, dividing by ρ_0 and eliminating $-(1/\rho_0)(\partial^2 p/\partial x\partial y)$ from eq. (7), we get

$$\beta g_y\frac{\partial(t-t_0)}{\partial x}=\nu\frac{\partial^3 u_1}{\partial y^3}-\frac{\sigma B_0^2}{\rho_0}\frac{\partial u_1}{\partial y}, \tag{8}$$

where $\nu = \mu/\rho_0$ is the kinematic viscosity.

Introducing the following non-dimensional quantities,

$$\xi_t = x/(a.Pe), \quad \eta = y/a, \ u = u_1/u_0, \qquad \dots \ (9)$$

into eq. (8), we have,

$$\frac{\beta a^2 g_y}{\nu U_0 P_e} \frac{\partial(t-t_0)}{\partial \xi_t} = -\frac{\sigma B_0 a^2}{\rho_0 \nu} \frac{\partial u}{\partial \eta} + \frac{\partial^3 u}{\partial \eta^3}. \qquad \dots \ (10)$$

Here a is the separation between the two plates and U_0 is the average velocity of the fluid. This eq. (10) will describe the fully-developed flow only when the right-hand side is independent of ξ. To satisfy this condition, we assume that the wall temperature varies linearly in the direction of the flow i.e., the heat flux at the wall is constant. Mathematically, this is represented by

$$t-t_0 = A_1\xi + T(\eta), \qquad \dots \ (11)$$

where A is the axial temperature gradient and $A_1 = Aa$. On substituting eq. (11), eq. (10) reduces to the following non-dimensional form

$$\frac{d^3 u}{d\eta^3} - M^2 \frac{du}{d\eta} = G, \qquad \dots \ (12)$$

where

$$G = \frac{\beta g_y a^2 A_2}{\nu u_0}, \quad \text{Grashof number}$$

$$A_2 = \frac{A_1}{P_e},$$

and $$M = B_0 a(\sigma/\rho_0 \nu)^{\frac{1}{2}},$$

the Hartmann number. The constant A_2 has the dimension of temperature. Also, the P'eclet number P_e is defined as the product of the Reynolds number $U_0 a/\nu$ and the Prandtl number ν/λ i.e., $\frac{U_0 a}{\lambda}$. Now A_1 in eq (11) may have the positive or negative values which physically corresponds to heating or cooling of the channel plates. Hence in terms of G, the heating or cooling of the plate is represented by

$$G > 0, \text{ (heating)}; \quad G < 0, \text{ (cooling)}.$$

This relation is useful for the physical interpretation of the results.

The no-slip boundary conditions are

$$u(0) = 0, \quad u(1) = 0, \qquad \dots \ (13)$$

As eq. (12) is of the third order, one more condition is necessary to solve it completely. So we employ the equation of continuity as the additional condition which, under no-slip conditions, may be stated as follows :

$$\int_0^1 u d\eta = 1. \tag{14}$$

The solution of eq. (12), in virtue of eqs. (13) and (14), is given by

$$u = a_1(\cosh M\eta - 1) + b_1 \sinh M\eta - G\eta/M^2, \tag{15}$$

where

$$a_1 = \frac{G[\sinh M(2\sinh M - M(\cosh M-1) - 4(\cosh M-1)]}{2M^2(\cosh M-1)(M\sinh M - 2(\cosh M-1)}$$

$$\frac{M\sinh M}{M\sinh M - 2(\cosh M-1)}$$

$$b_1 = \frac{M(\cosh M-1) + G[M(\cosh M+1) - 2\sinh M]/M^2.}{M\sinh M - 2(\cosh M-1)}$$

The velocity profiles, calculated from eq. (15), are plotted in figures 1 and 2 for positive and negative values of G. Once u is determined, we can now find the current density from eq. (3) which in non-dimensional form becomes

$$J = u - e, \tag{16}$$

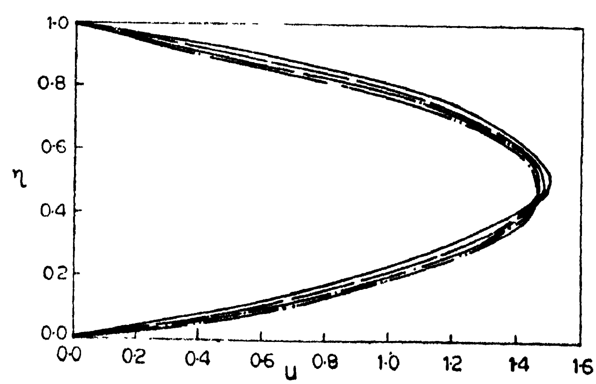

Fig. 1. Velocity profiles. G = −10—— —; 5— — —; $G = 0$—.—.—; 5—..—..—; M ·

where $J = j_z/\sigma U_0 B_0$ and $e = -E_z/U_0 B_0$. The current density from eq. (16) is plotted in figure 3. From Maxwell's equations, the magnetic field is given by,

$$J = -\frac{1}{R_m}\frac{dH}{d\eta}, \quad \ldots \tag{17}$$

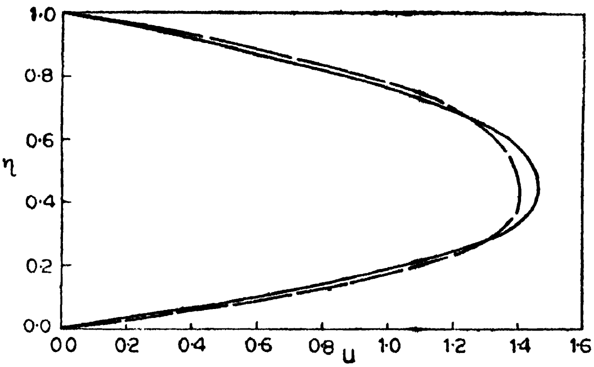

Fig. 2. Velocity profile. $G = 10$; $M = 2$—; 4— — —.

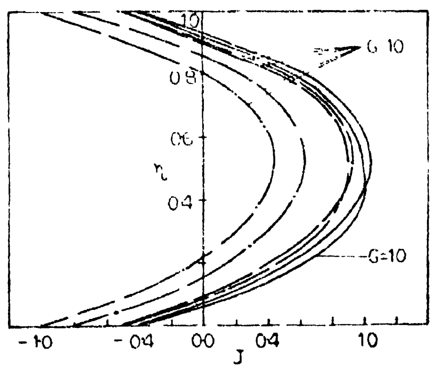

Fig. 3. Current density, M = 4, $e = 0.4$—; 0.5— —; 0.8—.—; 1.0—..—.

where $R_m = \mu_c \sigma U_0\, a$ is the magnetic Reynolds number and μ_c is the magnetic permeability of the fluid medium. To determine the induced magnetic field from eq. (17), knowledge of the geometry of the external circuit is necessary. Hughes & Young (1966) have described such a geometry which leads to the condition

$$H(0) = 0, \tag{18}$$

Hence from eqs. (15), (16), (17) and (18), we get

$$\frac{H}{R_m} = e\eta - \left[a_1\left(\frac{\sinh M\eta}{M} - \eta\right) + \frac{b_1}{M}(\cosh M\eta - 1) - \frac{G\eta^2}{2M^2}\right]. \quad \ldots \tag{19}$$

H/R_m is plotted in figure 4

The skin-friction, in non-dimensional form, is given by

$$\tau = -\frac{2}{R}\frac{du}{d\eta}\bigg|_{\eta=0} \quad \ldots \tag{20}$$

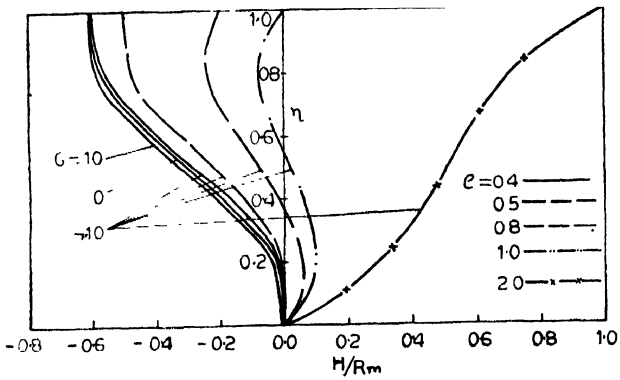

Fig. 4. Magnetic field, M = 2

Hence from eqs. (15) and (20), we have

$$\tau = -\frac{2}{R}(b_1 M - G/M^2). \qquad \ldots \quad (21)$$

The numerical values of $(-R\tau/2)$ are entered in table 1.

Table 1. Values of $R\tau/2$

$M \diagup G$	-10	-5	0	5	10
2	5.6065	5.9978	6.3891	6.7804	7.1716
4	6.7728	7.1086	7.4444	7.7802	8.1161

Knowing the velocity field, we can now determine the temperature field from eq. (5) which in view of eqs. (9), (11) reduces to the following non-dimensional form.

$$u = \frac{d^2\theta}{d\eta^2} + Br\left[\left(\frac{du}{d\eta}\right)^2 + M^2(u-e)^2\right] + Q, \qquad \ldots \quad (22)$$

where

$$\theta = \frac{T(\eta)}{P_e . A_2}$$

$$Br = \frac{\mu U_0^2}{\rho_0 c_p \lambda A_2 P_e}$$, Brinkman number

$$Q = \frac{Q_1 a^2}{\lambda A_2 P_e},$$ non-dimensional heat-source parameter.

The boundary conditions are (θ_w—upper wall temperature)

$$\theta(0)=0, \quad \theta(1)=\theta_w$$

or in terms of constant heat flux

$$\frac{d\theta}{d\eta}=-\frac{q(0)a}{KA_1}, \quad \frac{d\theta(1)}{d\eta}=-\frac{q(1)a}{KA_1}. \qquad \ldots \quad (23)$$

After substituting for u from eq. (15), the solution of eq. (22). in view of eq. (23) is as follows

$$\begin{aligned}
\theta = {} & \theta_w\eta+a_1\left[\frac{\eta-1+\cosh M\eta-\eta\cosh M}{M^2}-\frac{\eta^2-\eta}{2}\right]+ \\
& b_1\left[\frac{\sinh M\eta-\eta\sinh M}{M^2}\right]-\frac{G}{6M^2}(\eta^3-\eta)- \\
& -Br\left[\frac{C_2}{4M^2}(\cosh 2M\eta-\eta\cosh 2M+\eta-1)-\frac{C_3}{M^2}(\sinh M\eta-\eta\sinh M)-\right. \\
& -\frac{C_4}{M^2}(\cosh M\eta-\eta\cosh M+\eta-1)+\frac{D_1}{2}(\eta^2-\eta)+ \\
& +\frac{C_1}{6}(\eta^3-\eta)+\tfrac{1}{2}a_1b_1(\sinh 2M\eta-\eta\sinh 2M)+ \\
& +\frac{G^2}{12M^2}(\eta^4-\eta)-2a_1G\left\{\frac{\eta(\cosh M\eta-\cosh M)}{M^2}-\right. \\
& \left.-\frac{2(\sinh M\eta-\eta\sinh M)}{M^3}\right\}-2b_1G\left\{\frac{\eta(\sinh M\eta-\sinh M)}{M^2}-\right. \\
& \left.\left.-\frac{2(\cosh M\eta-\eta\cosh M+\eta-1)}{M^3}\right\}\right]-\frac{Q\eta}{2}(\eta-1)
\end{aligned} \qquad (24)$$

where $$\begin{aligned}
\theta_w = {} & Br\left[\frac{C_2-4C_4}{4M^2}(1-\cosh M)+\frac{a_1b_1}{2}(2M-\sinh 2M)\right. \\
& -\frac{C_3}{M^2}(M-\sinh M)-G^2/12M^2-\frac{C_1}{6}-\frac{D_1}{2} \\
& -2a_1G\{2\sinh M-M(\cosh M+1)\}/M^3-2b_1G\{M\sinh M \\
& +2(1-\cosh M)\}/M^3-1/\{1-q(1)/q(0)\}-a_1(M^2-2(\cosh M-1))/M^2 \\
& -b_1(M-\sinh M)/M^2-G/6M^2-Q/2
\end{aligned}$$

$C_1=2G(a_1+e)$,
$C_2=M^2(a_1{}^2+b_1{}^2)$,
$C_3=2C_1a_1/M+2b_1M^2(a_1+e)$,
$D_1=G^2/M^4+M^2(a_1+e)^2$.

The temperature profiles, calculated from eq. (24), are shown in figure 5. In technological fields, the rate of heat transfer is important and is defined in terms of Nusselt number as

$$Nu = -\frac{a}{A_2 P_e} \frac{d(T(\eta)-T(0))}{dy}\bigg|_{y=0} \qquad \frac{d\theta}{d\eta} \tag{25}$$

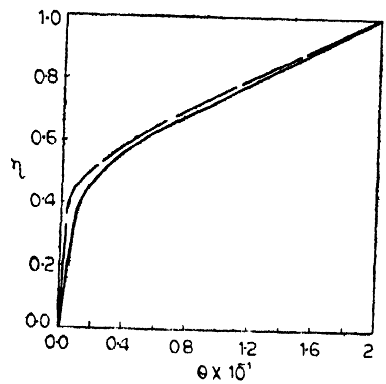

Fig. 5. Temperature profiles, $G = 5$; $M = 2$; Br $= 0.05$; $\epsilon = 0.4$—; 0.5— — — $Q = 0.2$.

On substituting for θ from eq. (24) in (25), Nu is calculated and its values are entered in table 2.

Table 2. Values of Nusselt number. $M = 2$, $Q = 0.2$, $B = 0.3$

e \ G	-10	-5	0	5	10
0	-0.2025	0.0862	0.4917	1.0141	1.6512
0.2	-0.1850	0.1049	0.5107	1.0323	1.6697
0.4	-0.1715	0.1187	0.5247	1.0466	1.6842
0.5	-0.1662	0.1240	0.5302	1.0522	1.6900
0.8	-0.1565	0.1342	0.5407	1.0631	1.7012
1.0	-0.1550	0.1359	0.5427	1.0654	1.7038
2.0	-0.2075	0.0846	0.4927	1.0166	1.6563

3. Variable Viscosity

Here we assume that the axial variation of viscosity is negligible and therefore we take into account the variation of μ with respect to η only. Under these conditions eq. (10),in virtue of eq. (11), may be written as

$$\frac{d^2}{d\eta^2}\left(\frac{\mu}{\mu_0}\frac{du}{d\eta}\right) - M^2 \frac{du}{d\eta} = G,$$

where the viscosity used in G is μ_0, the initial value of μ at the lower plate. We now introduce the transformation

$$u = \int_0^\varphi \frac{d\phi}{\mu/\mu_0}. \tag{27}$$

into eq (26), which then reduces it to

$$\frac{d^3\phi}{d\eta^3} - M^2 \frac{d\phi}{d\eta} = G. \tag{28}$$

For electrically conducting fluids, like liquid sodium, the viscosity can be approximated by,

$$\frac{\mu_0}{\mu} = \frac{t+t_a}{t_0+t_a} = 1 + \frac{t-t_0}{t_a+t_0} = 1+\epsilon\theta. \tag{29}$$

where, $\epsilon = A_2.P_e/(t_a+t_0)$. Here t_a is a reference temperature for the viscosity. Hence eq. (27), in virtue of eq. (29), reduces to

$$u' = \phi + \epsilon \int_0^\eta \theta\left(\frac{d\phi}{d\eta}\right) d\eta, \tag{30}$$

and on neglecting heat due to viscous, Joule dissipation and heat sources, the energy equation can now be written as,

$$\frac{d^2\theta'}{d\eta^2} = u^2 = \Phi + \epsilon \int_0^\eta \theta \left(\frac{d\Phi}{d\eta} \right) d\eta. \tag{31}$$

Here u' and θ' represent the velocity and temperature in case of variable viscosity. The eqs. (30) and (31) are now solved by the method of iteration. This is accomplished as follows. First eq. (28) is solved for Φ. This expression for Φ and the one for θ from eq. (24), are then substituted in eq. (30). The constants of integration are determined from the boundary conditions eqs. (13) and (14). Then this value of u' is integrated twice and after determining the constnts from eq.

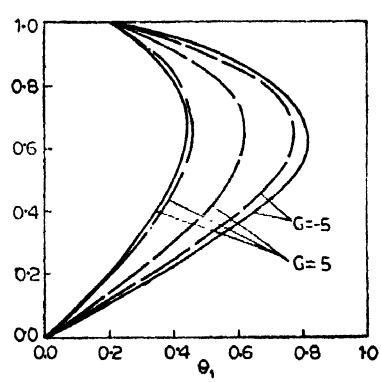

Fig. 6. Temperature profiles, $M = 2$, $\varepsilon = 0.4$—; ;.5— — —; 2— — —; Br = 0·05; $\epsilon = 0.2$,

(23), the expression for θ' is obtained and θ' is shown in figure 6. With this new expression for u', the skin-friction is calculated with the help of eq. (20) and its values are entered in table 3. Similarly from the expression for θ', the Nusselt number is also calculated and the values are entered in table 4.

Table 3. Values of $(-R_T/2)$
$M = 2$, $Q = 0.2$, $\theta_w = 0.2$, $\epsilon = 0.1$, $B_r = 0.03$

e \ G	−10	−5	0	5	10
0	−5.2165	2.0568	3.9906	3.7946	2.6354
0.2	−4.9510	2.0000	4.0285	3.8829	2.7592
0.4	−4.7119	1.9408	4.0648	3.9740	2.8870
0.5	−4.6016	1.9105	4.0823	4.0207	2.9526
0.8	−4.3006	1.8172	4.1320	4.1653	3.1560
1.0	−4.1230	1.7534	4.1726	4.2657	3.2977
2.0	−3.4399	1.4279	4.2825	4.8202	4.0947

Table 4. Values of $(-Nu)$,
$M = 2$, $Q = 0.2$, $\theta_w = 0.2$, $\epsilon = 0.1$, $B_r = 0.03$

e \ G	−10	−5	0	5	10
0	1.6652	3.0495	2.0824	2.3038	5.2128
0.2	1.7253	3.0601	2.1558	2.3486	5.2019
0.4	1.7757	3.0659	2.2290	2.3968	5.1940
0.5	1.7977	3.0670	2.2655	2.4222	5.1911
0.8	1.8520	3.0639	2.3744	1.5038	5.1877
1.0	1.8797	3.0567	2.4463	2.5629	5.1899
2.0	1.9416	2.9714	2.7918	2.9203	5.2667

7. Conclusion

In order to study the effects of different parameters, the values of e are chosen from the practical point of view. The efficiency of a mhd generator may be defined as the ratio of the electrical power to the flow power, which is identical to the value of the electric field factor e. But as pointed out by Moffatt (1963), the value of e for the maximum power is 0.5. In general, a reasonable compromise is, therefore, required between the conflicting requirements for maximum efficiency and the maximum power; $e = 0.8$ is the generally accepted value. Hence, one of the values of e is chosen as 0.8. Also, to bring out the effect of heating and cooling of the plate, the positive and negative values of G are also taken. We have the following observations :

1. *Constant viscosity*

From table 1, we conclude that with more heating of the plates, the skin-friction increases and with more cooling of the plates, the skin-friction decreases. With increasing M, the skin-friction also increases both in case of cooling or heating of the plates.

From table 2 we conclude that with more cooling of the plates, the rate of heat transfer decreases whereas with more heating of the plates, it increases. In a mhd generator, an increase in e, the loading parameter, leads to an increase in the Nu for all G.

2. *Variable viscosity*

The numerical values of the skin-friction are shown in table 3 for $M = 2$, $Q = 0.2$, $\theta_w = 0.2$ and $B_r = 0.03$. For all e, we observe that with more cooling of the plates, the numerical values of the skin-friction are negative. Hence separation may occur at the lower plate. The numerical values of $G \gtrless 0$ are always less than those for $G = 0$. Hence, when the viscosity is variable, the presence of buoyancy forces reduces the skin-friction. In case of a mhd generator, the skin-friction increases with incressing the loading parameter and when the plates are being heated. But when the plates are being cooled, for the same device, the skin-fricion decreases with an increase in e.

The numerical values of the Nusselt number are shown in table 4. In case of a mhd generator, the rate of heat transfer increases with increasing e, in case of both heating or cooling of the plates. With increasing G, it increases and decreases with decreasing G.

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